OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) 4763

Mechanics 3

Candidates answer on the answer booklet.

OCR supplied materials:
- 8 page answer booklet (sent with general stationery)
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Wednesday 26 January 2011
Afternoon

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet. Please write clearly and in capital letters.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by g m s^{-2}. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **8** pages. Any blank pages are indicated.

1 **(i)** Write down the dimensions of force, density and angular speed. **[3]**

The breaking stress, S, of a material is defined by

$$S = \frac{F}{A}$$

where F is the force required to break a specimen with cross-sectional area A.

(ii) Show that the dimensions of breaking stress are $\mathrm{ML^{-1}T^{-2}}$. **[2]**

In SI units (based on kilograms, metres and seconds), the unit of breaking stress is the pascal (Pa). The breaking stress of steel is 1.2×10^9 Pa.

(iii) Find the breaking stress of steel when expressed in a new system of units based on pounds, inches and milliseconds, where 1 pound = 0.454 kg, 1 inch = 0.0254 m and 1 millisecond = 0.001 s. **[3]**

A material has breaking stress S and density ρ. When a disc of radius r, made from this material, is rotated very quickly, there is a critical angular speed at which the disc will break apart. This critical angular speed, ω, is given by

$$\omega = kS^{\alpha}\rho^{\beta}r^{\gamma}$$

where k is a dimensionless constant.

(iv) Use dimensional analysis to find α, β and γ. **[4]**

Steel has breaking stress 1.2×10^9 Pa and density $7800\,\mathrm{kg\,m^{-3}}$. For a steel disc of radius 0.5 m the critical angular speed is $3140\,\mathrm{rad\,s^{-1}}$. Aluminium has density $2700\,\mathrm{kg\,m^{-3}}$ and for an aluminium disc of radius 0.2 m the critical angular speed is $8120\,\mathrm{rad\,s^{-1}}$.

(v) Find the breaking stress of aluminium. **[4]**

Using a different system of units, a disc of radius 15 is made from material with breaking stress 630 and density 70.

(vi) Find, in these units, the critical angular speed for this disc. **[2]**

2 **(a)** A particle P, of mass 48 kg, is moving in a horizontal circle of radius 8.4 m at a constant speed of V m s^{-1}, in contact with a smooth horizontal surface. A light inextensible rope of length 30 m connects P to a fixed point A which is vertically above the centre C of the circle, as shown in Fig. 2.1.

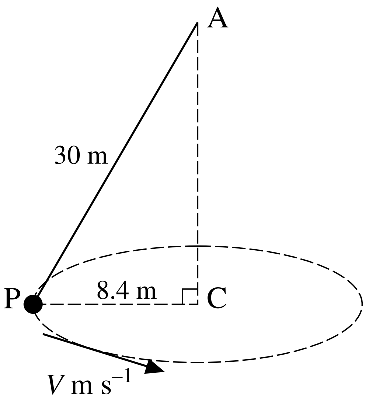

Fig. 2.1

(i) Given that $V = 3.5$, find the tension in the rope and the normal reaction of the surface on P. **[5]**

(ii) Calculate the value of V for which the normal reaction is zero. **[4]**

(b) The particle P, of mass 48 kg, is now placed on the highest point of a fixed solid sphere with centre O and radius 2.5 m. The surface of the sphere is smooth. The particle P is given an initial horizontal velocity of u m s^{-1}, and it then moves in part of a vertical circle with centre O and radius 2.5 m. When OP makes an angle θ with the upward vertical and P is still in contact with the surface of the sphere, P has speed v m s^{-1} and the normal reaction of the sphere on P is R N, as shown in Fig. 2.2.

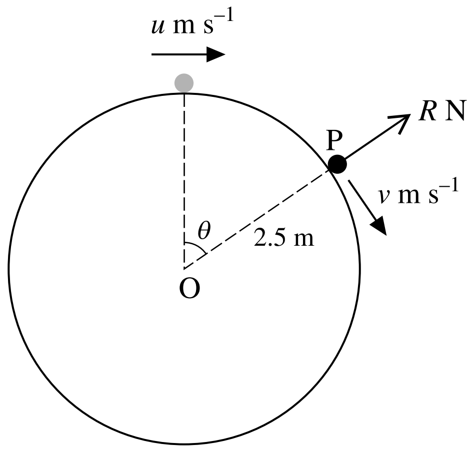

Fig. 2.2

(i) Show that $v^2 = u^2 + 49 - 49\cos\theta$. **[3]**

(ii) Find an expression for R in terms of u and v. **[4]**

(iii) Given that P loses contact with the surface of the sphere at the instant when its speed is 4.15 m s^{-1}, find the value of u. **[2]**

3 A block of mass 200 kg is connected to a horizontal ceiling by four identical light elastic ropes, each having natural length 7 m and stiffness $180\,\mathrm{N\,m^{-1}}$. It is also connected to the floor by a single light elastic rope having stiffness $80\,\mathrm{N\,m^{-1}}$. Throughout this question you may assume that all five ropes are stretched and vertical, and you may neglect air resistance.

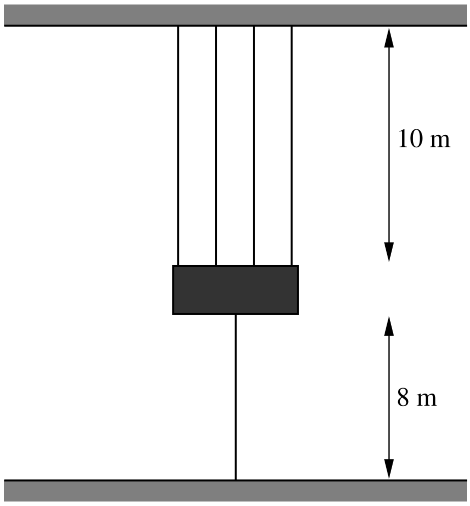

Fig. 3

Fig. 3 shows the block resting in equilibrium, with each of the top ropes having length 10 m and the bottom rope having length 8 m.

(i) Find the tension in one of the top ropes. **[2]**

(ii) Find the natural length of the bottom rope. **[4]**

The block now moves vertically up and down. At time t seconds, the block is x metres below its equilibrium position.

(iii) Show that $\dfrac{\mathrm{d}^2x}{\mathrm{d}t^2} = -4x$. **[4]**

The motion is started by pulling the block down 2.2 m below its equilibrium position and releasing it from rest. The block then executes simple harmonic motion with amplitude 2.2 m.

(iv) Find the maximum magnitude of the acceleration of the block. **[2]**

(v) Find the speed of the block when it has travelled 3.8 m from its starting point. **[2]**

(vi) Find the distance travelled by the block in the first 5 s. **[4]**

4 **(a)**

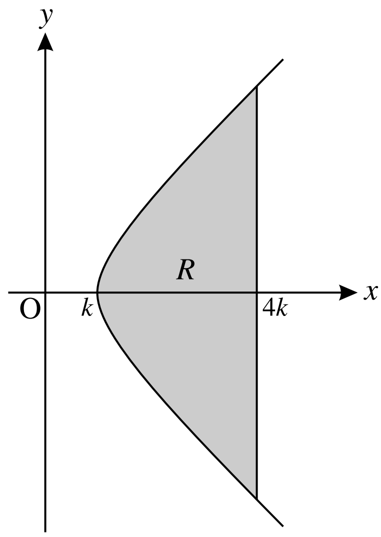

Fig. 4.1

The region R, shown in Fig. 4.1, is bounded by the curve $x^2 - y^2 = k^2$ for $k \leqslant x \leqslant 4k$ and the line $x = 4k$, where k is a positive constant. Find the x-coordinate of the centre of mass of the uniform solid of revolution formed when R is rotated about the x-axis. **[7]**

(b) A uniform lamina occupies the region bounded by the curve $y = \dfrac{x^3}{a^2}$ for $0 \leqslant x \leqslant 2a$, the x-axis and the line $x = 2a$, where a is a positive constant. The vertices of the lamina are O $(0, 0)$, A $(2a, 8a)$ and B $(2a, 0)$, as shown in Fig. 4.2.

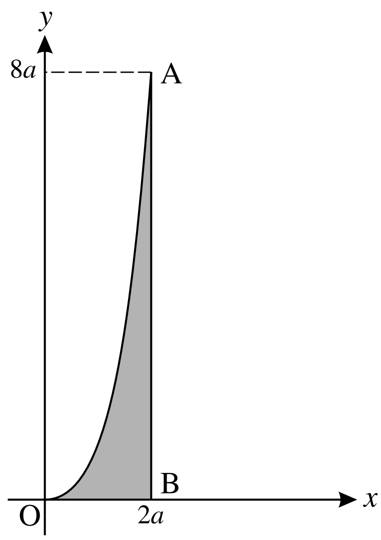

Fig. 4.2

(i) Find the coordinates of the centre of mass of the lamina. **[8]**

(ii) The lamina is freely suspended from the point A and hangs in equilibrium. Find the angle that AB makes with the vertical. **[3]**